Mathematics Honours Courses for 2007

Department of Mathematics – Macquarie University

First Half	**Second Half**
Applied Functional Analysis	Algebra
Lie Groups	Analysis
Topology	Elliptic Curves

First Half

Applied Functional Analysis

Lecturer: Paul Smith

Office: E7A 402

This unit prepares you to use differential and integral equations to attack significant problems in the physical sciences, engineering and applied mathematics. The concepts of functional analysis provide a suitable frameworkfor the development of effective analytical and computational methods to solve such problems. A selection of material will be drawn from the following topics.

1. Four alternative formulations of physical problems:conservation laws, boundary value problems, weak formulations and variational principles.
2. Green's functions and integral equations.
3. One dimensional boundary value problems and the Fredholm alternative.
4. Operators on Hilbert space and conditions for the solvability of equations.
5. Integral Equations.
6. Numerical Methods: Galerkin's method, least squares.
7. Ill-posed problems and their regularisation (stabilisation)
8. Effective treatments of potential theory problems and the scattering of waves by obstacles.

Prerequisites: MATH336 and MATH339

Lie Groups

Lecturer: Chris Meaney

Office: E7A 305

Lie groups are important in several branches of mathematics and physics since they provide the symmetries of structures. In this course we will focus on matrix groups and their Lie algebras. The aim is to describe the structure of the compact classical groups and begin to study their representation theory. The prerequisites are calculus in several variables, linear algebra, familiarity with groups, and some elementary topology.
Textbook: Kristopher Tapp, *Matrix groups for undergraduates*, Amer. Math. Society, 2005.

Topology

Lecturer: Ross Street and Thorsten Palm

Offices: E7A 311 (RS) and E7A 417 (TP)

Topology is the study of continuity. The definition of topological space was conceived in order to say what it means for a function between such spaces to be continuous. There are several different ways of defining topological structure and the proofs that these are equivalent abstract many concrete results about specific kinds of spaces. Different ways of expressing continuity are obtained. Sequences are not adequate for general topological spaces, they need to be replaced by nets or filters, and we discuss convergence of those. Particular properties

of topological spaces are analyzed in detail: these include separation properties, compactness, connectedness, countability conditions, local properties, metrizability, and so on. Applications to basic calculus are emphasized. A little bit of algebraic topology may be included by discussing the Poincaré or fundamental group of a space.
Prerequisite: MATH300

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Second Half

Algebra

Lecturers: Michael Batanin, Chris Cooper, Alexei Davydov
Offices: E7A 309 (MB and AD) and E7A 303 (CC)

This honours unit devotes approximately half of its time to ring theory and half to representation theory, over the complex numbers, for finite groups. The ring theory half provides a grounding in non-commutative ring theory leading, in a somewhat round-about way, to the Wedderburn Structure Theorem for semi-simple algebras. It does not take the most direct path but rather develops some general radical theory before focusing on the nil-radical. The Wedderburn theorem is applied to (classical) representation theory, establishing the orthogonality of irreducible characters that was taken as a given in the MATH337 introduction. The theory of characters is then extended, with a brief excursion into the theory of algebraic integers, to include such methods as inducing characters from subgroups. Finally this character theory is applied to show that groups of order $p^a.q^b$ are soluble. As well as gaining a good theoretical knowledge of representation theory students develop considerable skill in calculating character tables of groups with few normal subgroups, not so much for its own sake but as a way of integrating their knowledge of the theory.
Prerequisite: MATH337

Analysis

Lecturer: Xuan Duong
Office: E7A 307

This course presupposes MATH 339 (Real and Functional Analysis) as the background and aims to give students a solid foundation for further study in Pure and Applied Mathematics. Some topics are treated with certain depth. We will study Lebesgue integration, positive Borel measures, and the all important function spaces L^p. Then we will study the elementary Hilbert space theory and Banach space techniques. We also plan to discuss bounded and unbounded linear operators together with their spectral properties.
Prerequisite: MATH339
Text books: Rudin's *Real and Complex Analysis* (topics 1 – 5); Reed & Simon's *Functional Analysis* (topic 6).

Elliptic curves

Lecturer: Rod Yager
Office: E7A 404

Diophantine equations of the form $y^2 + a_1 xy + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6$ are known as Elliptic curves. In many ways, they are the most interesting diophantine equations, as they lie between linear equations and equations of conic sections, which, if they have solutions, have infinitely many rational solutions, and curves of higher genus which always have only a finite number of rational solutions.

Study of the structure of the set of rational points on an elliptic curve and their arithmetic properties have been of deep interest in Number Theory since before the time of Gauss, and was the essential ingredient to Wiles' celebrated proof of Fermats' Last Theorem. In this Honours course, we will explore some aspects of this rich area of study.
Prerequisite: MATH338