

MATH 338 Warm-up Exercises

1. Write down all the ways of inserting binary brackets in the list: $abcde$.
2. Write down all the monoids with no more than three elements.
3. In a finite group, show that there are an even number of elements x with $x^2 \neq 1$.
4. Show that a finite group of even order (cardinality) has an odd number (and hence at least one) element y with $y^2 = 1$ and $y \neq 1$.
5. If all elements x of a monoid satisfy $x^2 = 1$, show that the monoid is commutative.
6. If a and b are elements of a ring R , prove that $a(-b) = (-a)b = -(ab)$ and $(-a)(-b) = ab$.
7. Prove that every ring with 5 or 7 elements is commutative.
8. For the two quaternions $\mathbf{q} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{q}' = 1 + \mathbf{j} - 5\mathbf{k}$, find $\mathbf{q}\mathbf{q}'$ and $\mathbf{q}'\mathbf{q}$.
9. Let \mathbf{R} be the ring of real numbers. Let $f: \mathbf{R} \longrightarrow \mathbf{R}$ be defined by $f(x) = |x|$. Is f a ring morphism? Why?
10. Let $f: \mathbf{Z}_4 \longrightarrow \mathbf{Z}_2$ be defined by $f(0) = 0$, $f(1) = 1$, $f(2) = 0$, $f(3) = 1$. Is f a ring morphism? Why?
11. Let $f: \mathbf{Q} \longrightarrow \mathbf{Z}$ be the function from the rationals to the integers defined by $f\left(\frac{s}{t}\right) = s$ where s and t are integers with no common factors and t is positive. Is f a ring morphism? Why?

More questions will appear from time-to-time.