The Project

Due in class Friday 5 November.

Here are your two polynomials;

Student	Quartic	Quintic
Cheuk Lam Au	$x^4 - 10x^2 - 6$	$3x^5 - 7x^2 + 3$
Jennifer Jayne Barraclough	$x^4 - 6x^2 - 15$	$3x^5 - 13x^2 + 5$
Sarah Janice Bolger	$x^4 - 10x^2 - 18$	$5x^5 - 13x^2 + 3$
Simon Peter Joseph Byrne	$x^4 - 10x^2 - 5$	$5x^5 - 19x^2 + 5$
Richard Chambers	$x^4 - 10x^2 - 2$	$7x^5 - 11x^2 + 3$
James Ian Douglas	$x^4 - 2x^2 - 2$	$7x^5 - 17x^2 + 5$
Matthew Joseph Judd	$x^4 - 4x^2 - 18$	$3x^5 - 19x^2 + 3$
Chun Pong Kung	$x^4 - 6x^2 - 6$	$3x^5 - 17x^2 + 5$
Dominic Siu Chun Lo	$x^4 - 10x^2 - 15$	$5x^5 - 17x^2 + 5$
Fu Ken Ly	$x^4 - 6x^2 - 14$	$5x^5 - 17x^2 + 3$
Gareth Leo Manson	$x^4 - 8x^2 - 14$	$3x^5 - 11x^2 + 3$
Laura Elizabeth Mason	$x^4 - 8x^2 - 6$	$7x^5 - 13x^2 + 3$
John Phillips	$x^4 - 6x^2 - 10$	$3x^5 - 17x^2 + 3$
John Tu	$x^4 - 6x^2 - 3$	$5x^5 - 19x^2 + 3$

For the quartic, you are to prove it is irreducible over the rationals; find all of its zeros in $\mathbf{C}$, the complex numbers; and find generators for a splitting field $\mathbf{K}$ for the polynomial over $\mathbf{Q}$ — it is possible to find a real number α and a negative integer d such that $\mathbf{K} = \mathbf{Q}(\alpha, \sqrt{d})$. Then find the degree of $\mathbf{K}$ over $\mathbf{Q}$, and a basis for $\mathbf{K}$ over $\mathbf{Q}$; list all the $\mathbf{Q}$-automorphisms of $\mathbf{K}$ by describing their effect on the generators; name an abstract group to which $G = G(\mathbf{K}/\mathbf{Q})$ is isomorphic; draw the lattice of subgroups of G; draw the lattice of subfields of $\mathbf{K}$, explicitly indicating which subfield corresponds to which subgroup, and giving for each subfield a generator or set of generators for the field over $\mathbf{Q}$; indicate which of these subfields is a normal extension of $\mathbf{Q}$, and, for each such normal extension $\mathbf{E}$, compute $G(\mathbf{E}/\mathbf{Q})$.

For the quintic, you are to prove that it is irreducible, and then compute its Galois group (that is, the group $G(\mathbf{K}/\mathbf{Q})$, where $\mathbf{K}$ is a splitting field for the polynomial). Since you won't want to find the zeros of this polynomial, you won't be able to find $\mathbf{K}$ explicitly; but you can still find the Galois group (as an abstract group) by our methods.

The project will be judged on the correctness and relevance of the calculations and on the coherence and clarity of the discussion.